EFFICIENT SENSITIVITY ANALYSIS AND OPTIMIZATION OF A HELICOPTER ROTOR

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Aeroelastic Optimization Approach

Aeroelastic optimization of a system essentially consists of the determination of the optimum values of design variables which minimize the objective function and satisfy certain aeroelastic and geometric constraints. The process of aeroelastic optimization analysis is shown in Figure 1. To carry out aeroelastic optimization effectively, one needs a reliable analysis procedure to determine steady response and stability of a rotor system in forward flight. The rotor dynamic analysis used in the present study is developed inhouse at the University of Maryland and is based on finite elements in space and time [1,2,3]. The analysis consists of two major phases: vehicle trim and rotor steady response (coupled trim analysis), and aeroelastic stability of the blade. For a reduction of helicopter vibration, the optimization process requires the sensitivity derivatives of the objective function and aeroelastic stability constraints. For this, the derivatives of steady response, hub loads and blade stability roots are calculated using a direct analytical approach. An automated optimization procedure is developed by coupling the rotor dynamic analysis, design sensitivity analysis and constrained optimization code CONMIN [4].
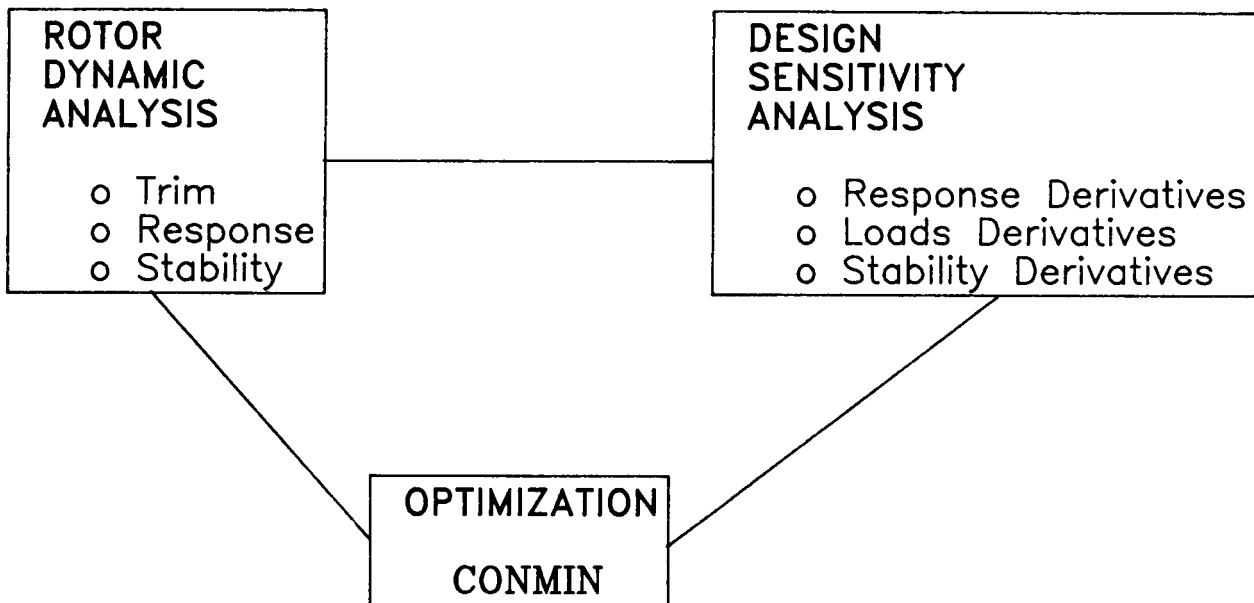

Figure 1

Coupled Trim Analysis

Coupled trim analysis in forward flight consists of calculation
of vehicle trim (propulsive), blade steady response and hub loads. The
vehicle trim solution determines the control settings and vehicle attitude
for the prescribed flight condition. It is calculated from the overall
nonlinear vehicle force and moment equilibrium equations. The blade
steady response solution involves the determination of time dependent
blade deflections at different azimuth locations. The blade is assumed
as an elastic beam undergoing flap bending, lag bending, elastic twist
and axial deflections, and is discretized into a number of beam elements.
To reduce computation time, a large number of finite-element equations
are transformed to a few (typically eight) normal mode equations. These
nonlinear periodic equations are then solved for steady response using
a finite-element method in time formulated from Hamilton's weak principle.
The hub loads are obtained using a force summation approach. For the
coupled trim analysis, the vehicle trim and rotor response equations
are solved iteratively as one coupled solution using a modified Newton
method. The converged trim and response solutions satisfy simultaneously
the overall force and moment equations of the vehicle. Figure 2 shows
the blade steady flap response at tip for an advance ratio of 0.3. For
a completely trimmed condition, there is no unbalanced force or moment
acting on the hub, and the lag and torsion responses consist primarily
of 1/rev amplitudes, whereas the flap response is dominated by 2/rev
amplitude.

BLADE FLAP RESPONSE

$C_T/\sigma = 0.07$, $\mu = 0.3$, $X_{CG} = 0.0$, $Y_{CG} = 0.0$, $h/R = 0.2$
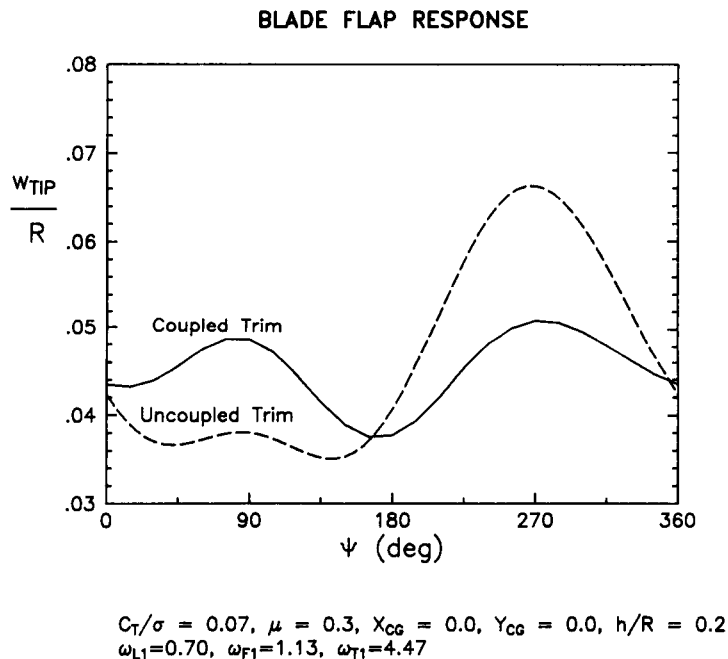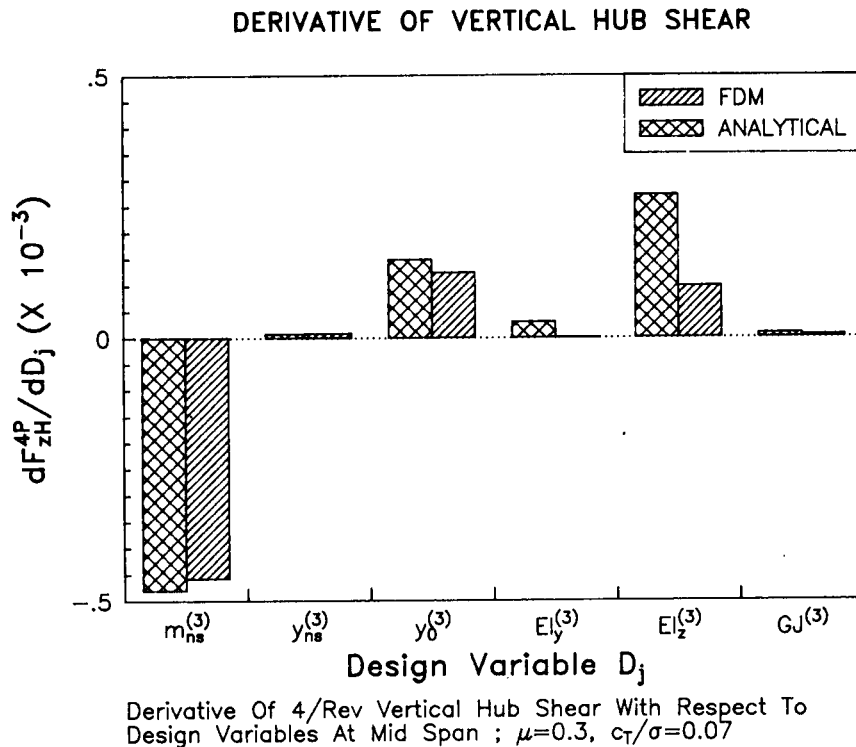$\omega_{L1}=0.70$, $\omega_{F1}=1.13$, $\omega_{T1}=4.47$

Figure 2

Design Sensitivity Analysis

A design sensitivity analysis involves calculation of sensitivity derivatives of the objective function and behavior constraints. Most of the optimization studies use finite difference approach to calculate sensitivity derivatives. This approach is easy to implement, but costly because of heavy computation time. Also, the selection of proper step size is not easy. However, a direct analytical approach is more complicated in formulation but reduces the computation time substantially. In the present study, the derivatives of blade response, hub loads and blade stability with respect to the design variables are calculated using a direct analytical approach [1,2,5]. The formulation of the derivatives of blade response including hub loads is developed as an integral part of the basic steady response analysis. The implementation of this scheme is made possible through the use of the finite-element method in time. Figure 3 compares the sensitivity derivatives of the 4/rev vertical hub shear with respect to the design variables at the mid span location. The numerical results for finite difference and direct analytical approaches show quite identical trends.

DERIVATIVE OF VERTICAL HUB SHEAR

Derivative Of 4/Rev Vertical Hub Shear With Respect To Design Variables At Mid Span ; μ=0.3, c_T/σ=0.07

Figure 3

CPU Time for Design Sensitivity Analysis

The stability sensitivity analysis involves the calculation of the derivatives of blade stability roots, and again constitutes an integral part of the basic stability analysis. For this, the Floquet transition matrix is extended to include the derivatives of blade stability roots. Figure 4 shows CPU time required in UNISYS-1100/90 for calculation of sensitivity derivatives of blade response, oscillatory hub loads (objective function) and blade dampings (behavior constraints) of the baseline blade using finite difference and direct analytical approaches. For five design variables, the CPU time used is 110 min for the finite difference, and 25 min for the direct analytical approach. For thirty design variables, the CPU time is increased to 560 min for the finite difference, while it is 50 min for the direct analytical approach. As the number of design variables is increased, the difference of the CPU time required becomes larger.
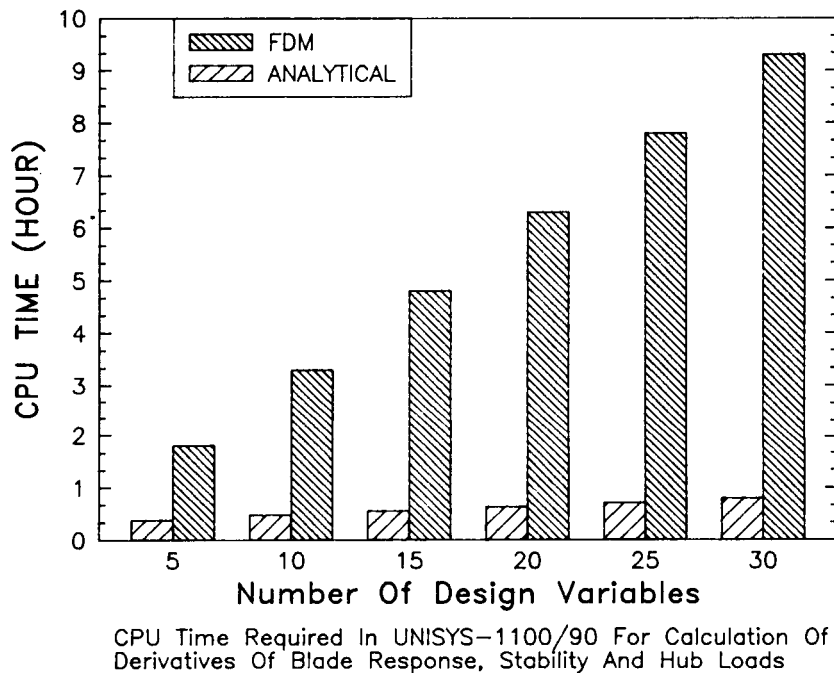

CPU Time Required In UNISYS-1100/90 For Calculation Of Derivatives Of Blade Response, Stability And Hub Loads

Figure 4

Design Variables

Figure 5 shows the blade and airfoil section. For the analysis, the blade is discretized into five beam elements of equal length, and the numerals indicate the order of beam elements. Each beam element consists of fifteen degrees of freedom, representing flap bending, lag bending, elastic twist and axial deflections. In the airfoil, the 'e.a.' denotes the elastic axis, and the m_0 is a baseline blade mass per unit length (reference), which has an offset of y_0. There can be placed an extra nonstructural mass (m_{ns}) at a chordwise location of y_{ns}. Therefore, structural design parameters can be chosen from nonstructural mass (m_{ns}), chordwise offset of nonstructural mass (y_{ns}), blade center of gravity offset (y_0), and blade flap bending stiffness (EI_y), lag bending stiffness (EI_z) and torsional stiffness (GJ). These structural parameters can have spanwise variations. Thus, the total design variables for five beam elements are

(6 structural parameters) × (5 beam elements) = 30
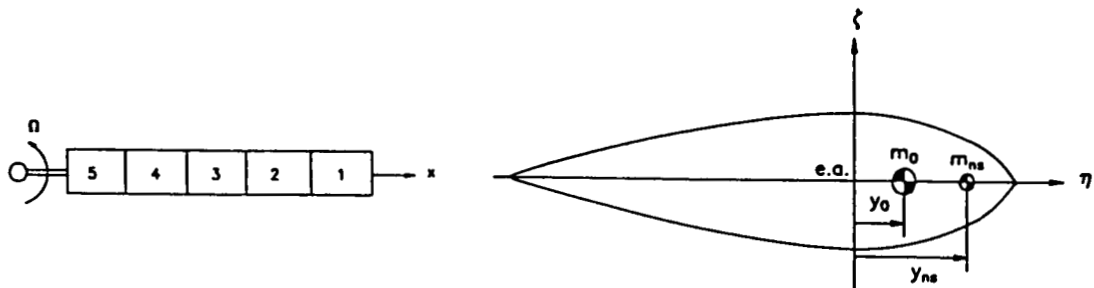

o   Nonstructural Mass

o   Chordwise Location of Nonstructural Mass

o   Chordwise Location of Blade CG

o   Blade Flap Bending Stiffness

o   Blade Lag Bending Stiffness

o   Blade Torsional Stiffness

* Spanwise Variations

* Total 30 Design Variables

Figure 5

Minimization of 4/Rev Vertical Shear Alone

Helicopter vibrations are characterized by means of oscillatory hub loads including three forces and three moments. To reduce helicopter vibrations, most of the optimization studies minimized 4/rev vertical shear alone for a four-bladed rotor, without constraining other components of oscillatory hub forces or moments. Figure 6 shows the optimization iteration history when 4/rev vertical hub shear alone is minimized. After 7 iterations, the 4/rev vertical hub shear is reduced by 75%. Other 4/rev hub loads are increased instead; an increase by 30% for longitudinal and lateral hub shears, 10% for rolling and lateral hub moments and 210% for yawing hub moment. This is due to the fact that other components of oscillatory hub loads besides 4/rev vertical hub shear are not involved in the objective function. This shows that one needs to make a careful choice of the objective function to achieve an optimum solution.

OPTIMIZATION ITERATION HISTORY
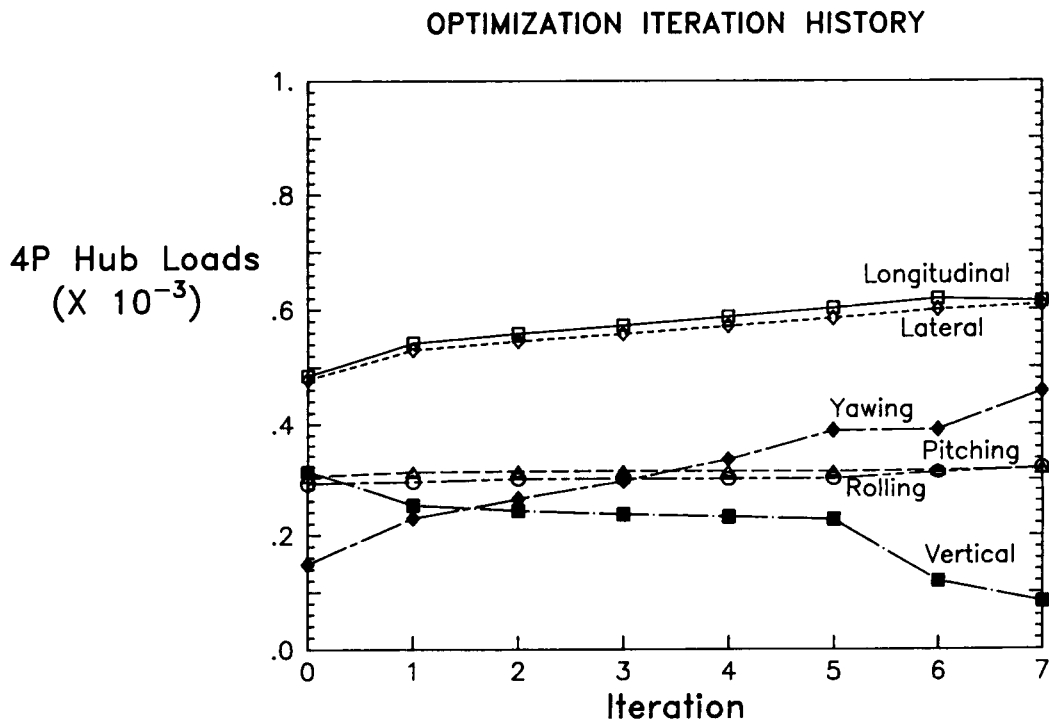
OBJECTIVE : MINIMIZATION OF 4/REV VERTICAL HUB SHEAR ALONE

Figure 6

Minimization of All Hub Forces and Moments

    The objective function involves all six components of hub forces
and moments in either the hub-fixed nonrotating frame or rotating frame,
and is defined as a sum of hub force resultant and moment resultant.
In the present study, hub loads in the nonrotating frame are used. The
weighting functions are simply chosen as unity. To achieve an optimum
solution, the best choice of design variables is found in Ref. [6] involving
nonstructural masses and their locations (chordwise and spanwise), and
spanwise distribution of blade flap bending, lag bending and torsional
stiffnesses. In this case, twenty five design variables are involved.
Figure 7 shows the optimization iteration history of the objective function.
Each optimization iteration involves updating the search direction from
the sensitivity analysis, determining the optimum move parameter by
polynomial approximation in the one dimensional search and checking
the convergence to terminate the optimization process. After each optimization
iteration, the objective function becomes reduced. The optimum solution
is obtained after 8 iterations, and a 77% reduction of the objective
function is achieved.

OPTIMIZATION ITERATION HISTORY

OBJECTIVE FUNCTION (X 10^{-3})

Iteration
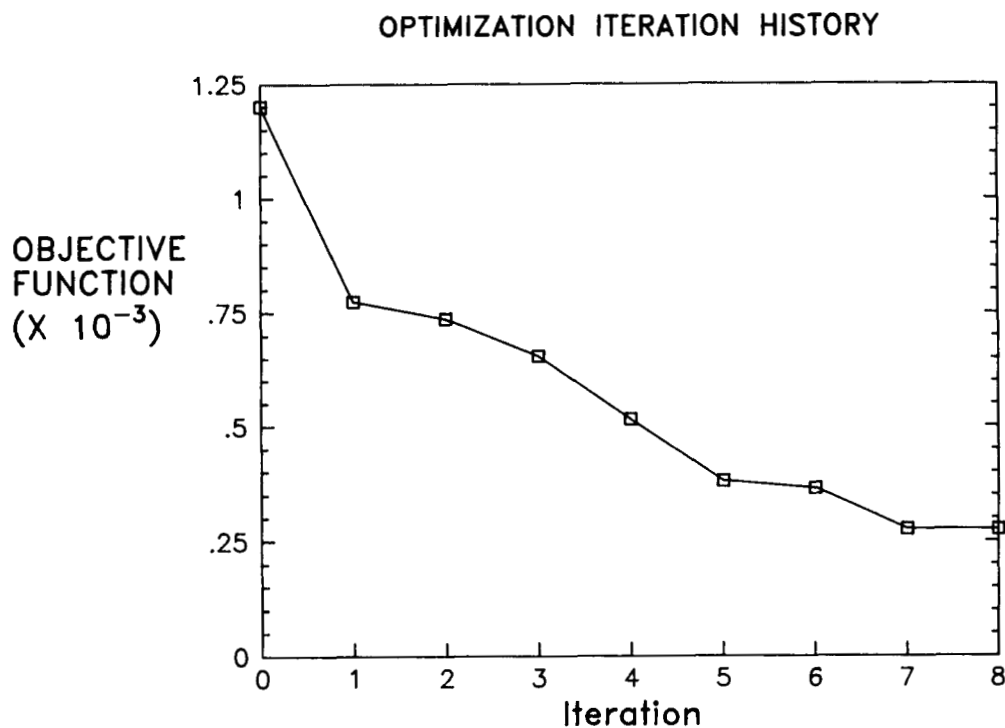
OBJECTIVE : MINIMIZATION OF ALL HUB FORCES AND MOMENTS

Figure 7

Optimum Hub Loads

Figure 8 compares optimum 4/rev hub forces and moments with the baseline values. The optimum result shows that all the 4/rev hub forces and moments are reduced from the baseline values. This is because all the components are included in the objective function, and also equal weighting function is enforced on each component. There are considerable reductions of 4/rev hub loads achieved: an 80% reduction for longitudinal and lateral hub shears, a 60% reduction for vertical hub shear, an 80% reduction for rolling and pitching hub moments and a 90% reduction for yawing hub moment. For a reduction of helicopter vibration, the objective function must, therefore, include all six components of 4/rev hub loads in conjunction with appropriate weighting functions.
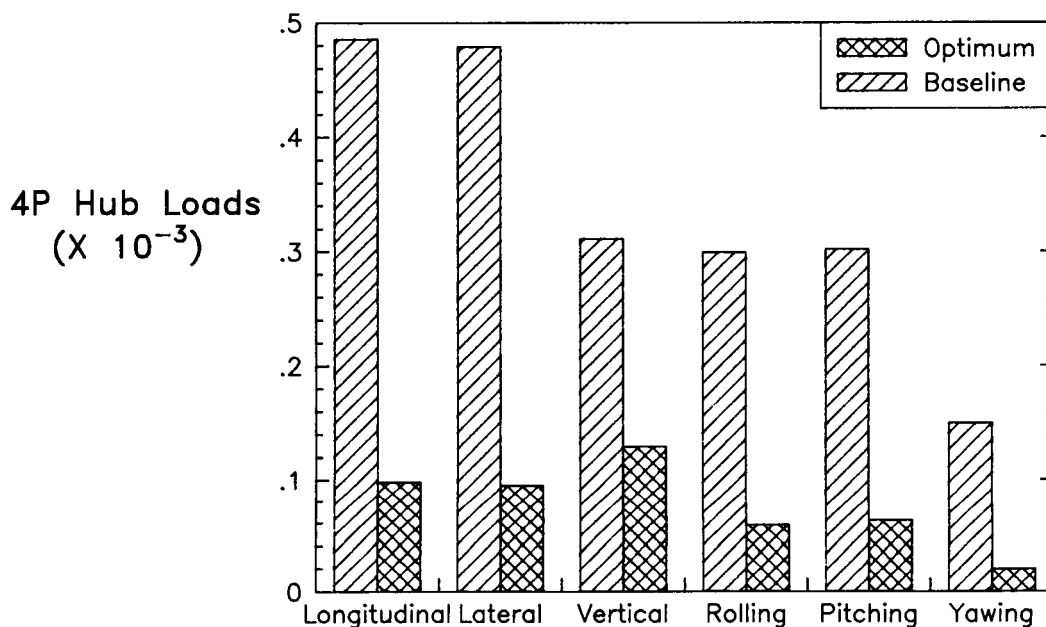
OBJECTIVE :   MINIMIZATION OF ALL HUB FORCES AND MOMENTS

Figure 8

Aeroelastic Stability Constraints

For structural optimization problems, one may impose behavior constraints which must be satisfied for a feasible design. In the present optimization analysis, the aeroelastic stability of the blade in forward flight is constrained to be stable for all modes. For this, the blade damping, which is the real part of the characteristic exponent with a negative sign, is kept in the positive range. Figure 9 shows the optimization iteration history of blade damping of first lag, flap and torsion modes. For lag and flap modes, the blade damping varies smoothly at each iteration. However, for torsion mode the damping is changed abruptly between iterations 2 and 4. This may be associated with a large shift of effective c.g. offset because of nonstructural masses. All three blade modes, however, remain stable for all iterations. Thus, the design solution in the optimization process stays within the feasible design space for all iterations (unconstrained optimization process).
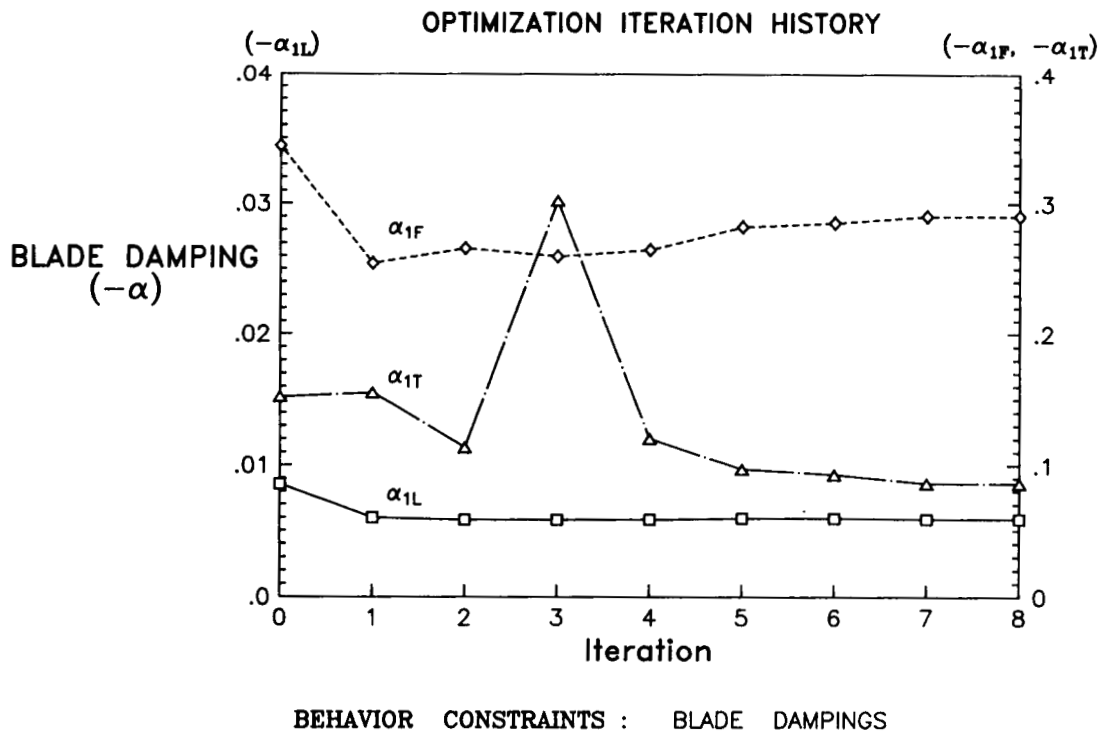

BEHAVIOR CONSTRAINTS : BLADE DAMPINGS

Figure 9

CPU Time for Optimization Analysis

Figure 10 shows the comparison of CPU time required for the optimization process on UNISYS 1100/90 using finite difference and direct analytical approaches. For finite difference approach, the CPU time is approximated based on the number of function evaluations. To achieve an optimum solution, there is about an 80% reduction in CPU time with the present approach as compared with the frequently adopted finite-difference approach. Comparing the CPU time for the sensitivity analysis, one can easily realize that this substantial reduction of CPU time results from an efficient evaluation of sensitivity derivatives of the objective function and/or constraints in the sensitivity analysis by using a direct analytical approach.
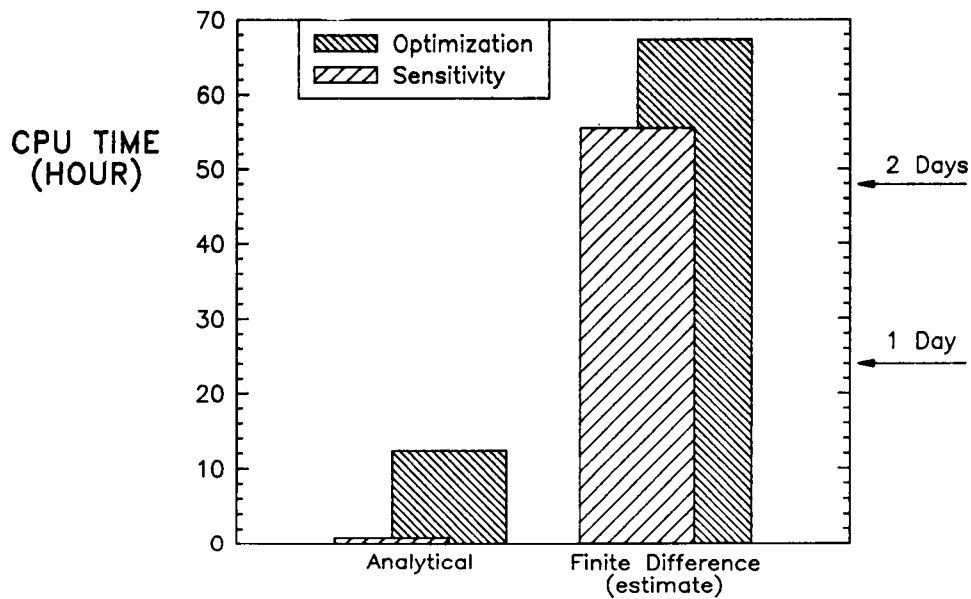

Comparison of CPU Time for Optimization Process in UNISYS 1100/90 Using Analytical and Finite Difference Approaches

Figure 10

Behavior Constraints -- Initially Infeasible

If the design solution stays in the feasible design space for all iterations, behavior (aeroelastic stability) constraints do not become active (see Figure 9). Here, we have investigated a case in which behavior constraints have been violated, right from the beginning for the baseline configuration. Figure 11 shows the optimization iteration history of blade damping of first lag, flap and torsion modes when 1% margin of blade damping is imposed for stability. The lag mode damping for the baseline configuration is less than 1%. In the next iteration, the design solution is moved into the feasible design space along the feasible direction by the optimizer CONMIN [4], and the blade becomes aeroelastically stable. In subsequent iterations, the blade stability is well maintained. Similar to Figure 9, the blade damping of lag and flap modes varies smoothly at each iteration, but the torsion mode damping is changed abruptly due to a large shift of effective c.g., offset resulted from the nonstructural mass placement.


OPTIMIZATION ITERATION HISTORY
INITIALLY INFEASIBLE
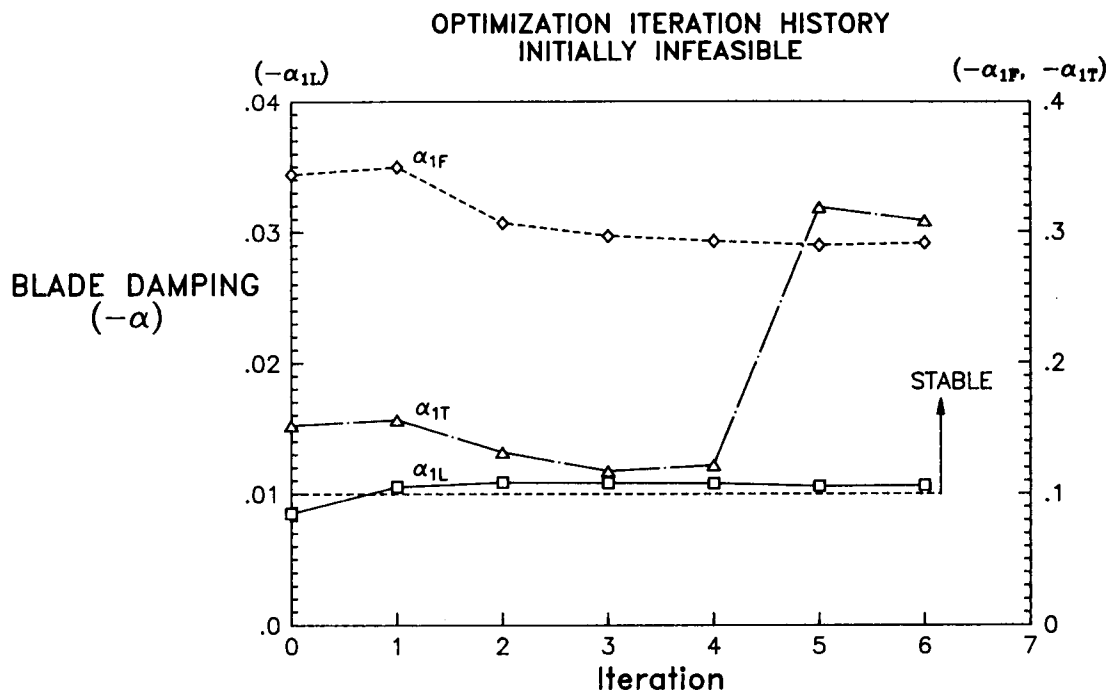
BEHAVIOR CONSTRAINTS :   BLADE DAMPINGS

** 1% DAMPINGS MAINTAINED FOR STABILITY

Figure 11

Initially Infeasible Design

Figure 12 shows the optimization iteration history of the objective function for the case in which behavior (aeroelastic stability) constraint is violated by the baseline configuration. The objective function involves minimization of all six components of 4/rev hub loads for a four-bladed rotor. The design variables involve nonstructural masses and their locations (spanwise and chordwise), and spanwise distribution of blade bending stiffnesses (flap, lag and torsion), and there are total twenty five design variables. The optimizer enforces the design solution to move along the feasible direction so that no behavior constraint is violated. After first iteration, the design solution becomes feasible (see Figure 11), and the objective function is slightly increased. In subsequent iterations, the objective function becomes continually reduced. The optimum solution is obtained after six iterations, and there is about a 25% reduction of the objective function achieved. Comparing with the case of initially feasible design where no stability constraint was violated and a reduction of 77% of the objective function was achieved, the optimum for initially infeasible design is far less achieved.

OPTIMIZATION ITERATION HISTORY
INITIALLY INFEASIBLE
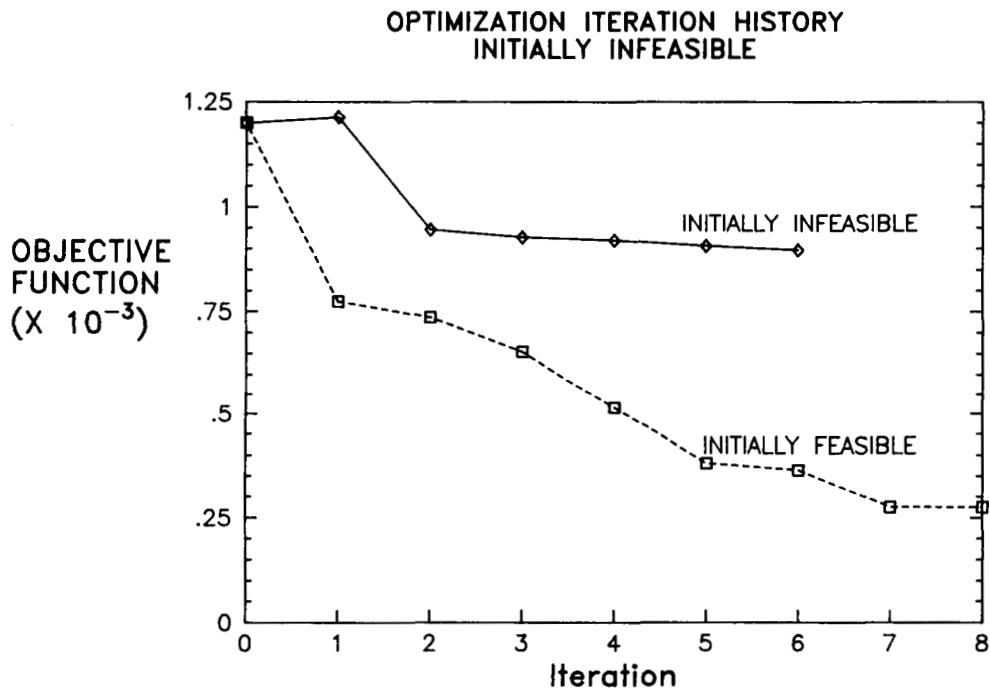
OBJECTIVE : MINIMIZATION OF ALL HUB FORCES AND MOMENTS

Figure 12

References

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[2] J. W. Lim, ''Aeroelastic Optimization of a Helicopter Rotor,'' Ph.D. Dissertation, University of Maryland, May 1988

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[4] G. N. Vanderplaats,''CONMIN - A Fortran Program for Constrained Function Minimization,'' User's Manual, NASA TMX 62282, 1973

[5] J. W. Lim and I. Chopra,''Stability Sensitivity Analysis for Aeroelastic Optimization of a Helicopter Rotor,'' 29th AIAA SDM Conference, Williamsburg, VA, April 1988 ; Also, Appeared in The Workshop on Dynamics and Aeroelastic Stability Modeling of Rotor Systems, Florida Atlantic University, November 1987

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